Testing of Cylinders in Sheared Flow

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It is desirable to develop a simple analytic method for the prediction of the drag of bodies in the boundary layer and for the design optimization of fairings and other bodies immersed in the boundary layer. This problem has been attacked by describing the sheared flowfield in terms of Poisson's equation, $\nabla^2\psi = -\zeta$. Poisson's equation has been solved using a method of finite differences and by direct solution. These solutions which are basically perfect fluid methods, have been compared with the results of experiments conducted in a small wind tunnel in which the sheared flow was produced by means of contoured honeycomb blocks. The comparison indicates good agreement except for regions of separated flows.

Nomenclature

a = distance between adjacent nodes on relaxation grid
A = cylinder radius
c = cylinder chord
C_L = coefficient of lift, $L'/\frac{1}{2}\rho V_0^2 c$
C_p = coefficient of pressure, $(p - p_0)/\frac{1}{2}\rho V_0^2$
d = honeycomb cell diameter
f = friction factor
K = constant, magnitude indicates amount of linear shear
L' = lift per unit span
$L(y)$ = length of honeycomb block
p = pressure
r = radial coordinate
Re = Reynolds number
u = x-component of velocity
v = y-component of velocity
V_0 = reference onset velocity
$V(y)$ = velocity downstream of honeycomb block
ψ = stream function
ρ = fluid density
θ = angular coordinate
ζ = vorticity (circulation per unit area)
∇ = del operator

Introduction

PREDICTING drag of the various protuberances which have been proposed to be added to the surfaces of an aircraft is a continuing problem. These protuberances include radar, communication, navigation, and weapons systems which involve sensors, antenna, radar housings, pylons, windows, canopies, and other bodies to be mounted on the aircraft. They usually extend beyond the smooth skin contours, but are almost always submerged in the boundary layer. Each proposed addition requires that the aerodynamicists estimate the drag penalty and conduct performance trades to arrive at suitable design.

The desire to improve the design method was the impetus for the investigations reported in this paper. It would be useful to have a simple analytic method to predict the drag of a given shape. The next step would be to devise a system to optimize, for low drag, the shape of a fairing or enclosure of given size within the boundary layer. Such a program may well be extended to other sheared flows of interest to airplane designers, such as jet engine jets, propeller slipstreams, and wakes behind wings and fuselages.

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Sheared Flowfield as Poisson's Equation

The initial analytical approach to the problem involved making two assumptions: 1) The body was placed in a sheared flowfield having an arbitrarily assumed velocity profile. There was no consideration of how the sheared flowfield was produced. 2) The vorticity of the field was assumed constant, i.e., there was no addition, or deterioration of vorticity, in the region under consideration. If these assumptions are made, the flowfield may be represented by the equation: $\nabla^2\Delta = -\zeta$. This form of equation is called Poisson's equation, it may be contrasted with Laplace's equation; $\nabla^2\psi = 0$.

Solution by Relaxation

Laplace's equation lends itself to solution by the relaxation technique of the method of finite differences. Monical[1] suggested that this method could be modified to be used to solve Poisson's equation.

To apply the method of finite differences to solution of Laplace's equation, a grid is superimposed on the flowfield (as in Fig. 1a). Initial values of the stream function are assigned to each node on the grid. These initial estimates are refined by successively relaxing each node and calculating a new value of ψ from the surrounding nodal values. If the star at a node is as shown in Fig. 1b, the formula for calculation of ψ is

$$\psi_0 = \frac{\dfrac{[(\psi_1/a_1) + (\psi_3/a_3)]}{a_1 + a_3} + \dfrac{[(\psi_2/a_2) + (\psi_4/a_4)]}{a_2 + a_4}}{\dfrac{1}{a_1a_3} + \dfrac{1}{a_2a_4}}$$

This solution of Laplace's equation may be modified for the sheared flowfield. The defining equation is a form of

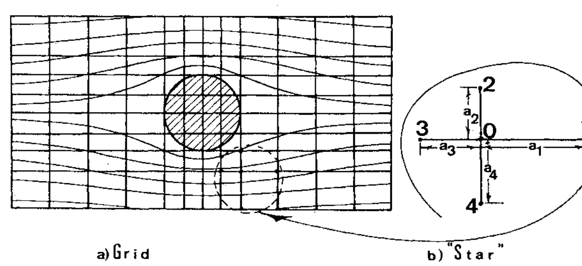

Fig. 1 Finite differences grid overlaid on flowfield.

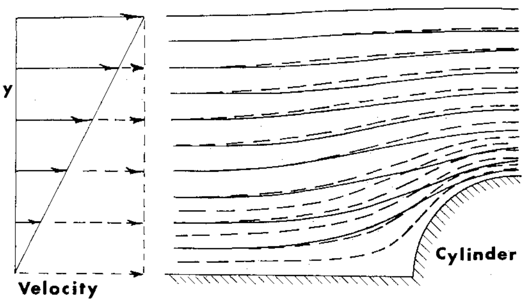

Fig. 2 Streamline pattern around a circular cylinder in linearly sheared flow.

Poisson's equation

$$\nabla^2\psi = -\zeta$$

$$\zeta = -[(\partial^2\psi/\partial x^2) + (\partial^2\psi/\partial y^2)]$$

This relation may be reduced[2] to

$$\psi_0 = \frac{\dfrac{[(\psi_1/a_1) + (\psi_3/a_3)]}{a_1 + a_3} + \dfrac{[(\psi_2/a_2) + (\psi_4/a_4)]}{a_2 + a_4} + \dfrac{\zeta}{2}}{\dfrac{1}{a_1a_3} + \dfrac{1}{a_2a_4}}$$

for a nodal value, as in Fig. 1b.

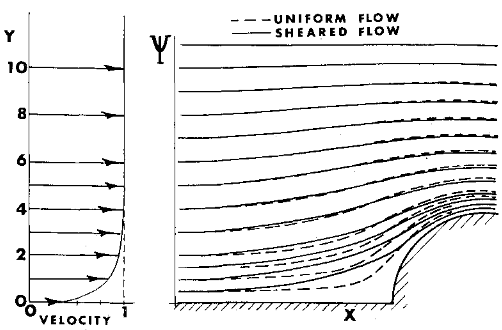

Fig. 3 Streamline pattern in typical boundary-layer flow.

If $a_1 = a_2 = a_3 = a_4$,

$$\psi_0 = \tfrac{1}{4}[(\psi_1 + \psi_2 + \psi_3 + \psi_4) + a^2\zeta]$$

While this solution is general, there are practical limitations to its use. In the case in which the onset flow is linearly sheared, i.e., the vorticity is constant throughout the field, the programming is simple and straightforward, and the solution is easily obtained. Figure 2 represents a streamline pattern obtained by this method.

In the case of sheared flowfields having other than a linear variation of velocity, there is a variation of vorticity through the field. Helmholtz' principle may be extended to state that the vorticity is constant along any streamline, i.e., vorticity is a function of stream function $[\zeta = \zeta(\psi)]$. This

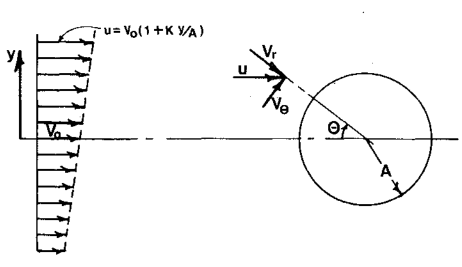

Fig. 4 Nomenclature.

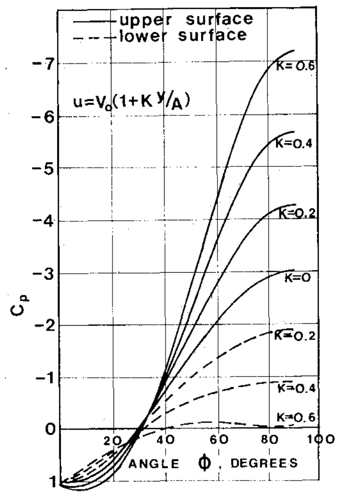

Fig. 5 Pressure distributions on a circular cylinder in linearly sheared flows.

relation changes equation for ψ_0 to the form

$$\psi_0 = \frac{\dfrac{[(\psi_1/a_1) + (\psi_3/a_3)]}{a_1 + a_3} + \dfrac{[(\psi_2/a_2) + (\psi_4/a_4)]}{a_2 + a_4} + \dfrac{1}{2}\zeta(\psi_0)}{\dfrac{1}{a_1a_3} + \dfrac{1}{a_2a_4}}$$

To attempt to solve this equation by relaxation involves

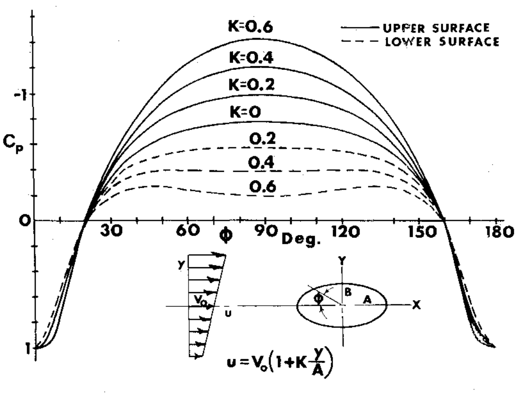

Fig. 6 Pressure distributions on a 3–1 elliptical cylinder in linearly sheared flows.

double approximation which makes the method impractical except for special cases.

One of these special cases is the case of an object attached to a wall boundary-layer type flow. The wall provides a boundary condition by spatially fixing the stagnation streamline. Figure 3 shows the type of streamline pattern which may be obtained for this case.

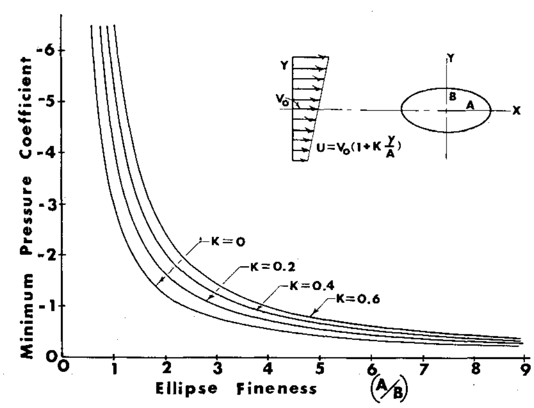

Fig. 7 Effect of linear shear on minimum pressure coefficients.

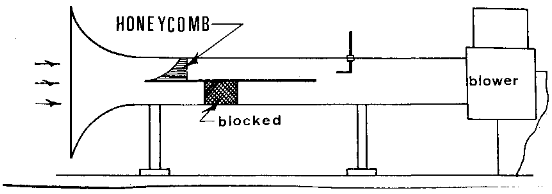

Fig. 8 Use of contoured honeycomb to produce turbulent boundary-layer profile in wind tunnel.

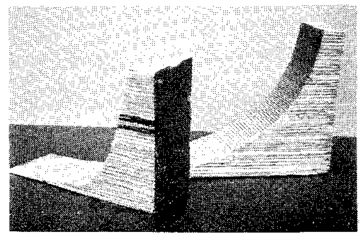

Fig. 10 Contoured honeycomb blocks.

Direct Solution

In order to attain a more practical method of solution for the general case of the sheared flowfield, Frutiger[3] developed a method of analysis based on dividing the stream function into two functions $\psi = \psi_0 + \psi_1$. ψ_0 is the stream function which represents the flowfield far from the surface of the body and ψ_1 is the disturbance function. The total vorticity is contained in ψ_0, and,

$$\nabla^2\psi_0 + \nabla^2\psi_1 = f(\psi_0)$$

The function ψ_1 must be found such that it satisfies the following conditions: 1) ψ_1 must vanish far from the body, 2) the normal velocity component due to ψ_1 at the surface must cancel the normal component due to ψ_0 at the surface, and 3) $\nabla^2\psi_1 = 0$.

Table 1 Lift developed by cylinders in sheared flow

Body	Velocity equation	C_L
Circle	$u = V_0$	0.00
Circle	$u = V_0(1 + 0.4y)$	1.47
Circle	$u = V_0(1 + 0.6y)$	2.80
3–1 Ellipse	$u = V_0(1 + 0.6y)$	2.43
5–1 Ellipse	$u = V_0(1 + 0.6y)$	1.41

Figure 4 shows a circular cylinder in linearly sheared flow with the onset velocity given by

$$u = V_0(1 + Ky/A)$$
$$v = 0$$

The vorticity far from the cylinder is given as

$$\zeta = (\partial v/\partial x) - (\partial u/\partial y) = -V_0K/A$$

This analysis, and the conditions for ψ_1, produce the equation for the total stream function

$$\psi = V_0\left[\left(r - \frac{A^2}{4r}\right)\sin\theta + K/2\left(\frac{V^2}{A}\sin2\theta + \frac{A^3}{32r^2}\cos\theta\right)\right.$$

and at the surface of the cylinder

$$\psi]_{r=A} = V_0KA/16$$

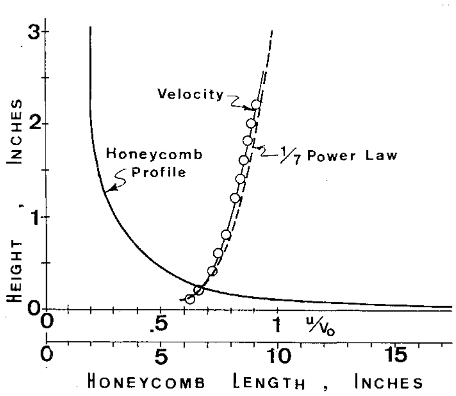

Fig. 9 Honeycomb contour and resulting velocity variation.

which shows that, at the surface of the cylinder, ψ is a constant and the cylinder surface is a (boundary) streamline.

In order to predict pressure variation, Euler's equations for two-dimensional flow are applied to this flow pattern. The pressure coefficient is

$$C_p = 1 - [(u^2 + v^2)/V_0^2] + 2K/A(\psi/V_0)$$

and, in polar coordinates, it is

$$C_p = 1 - (V_\theta/V_0)^2 + K^2/8$$

Note that in uniform flow, for which $K = 0$, this equation reduces to the familiar expression for a cylinder in uniform flow

$$C_p = 1 - 4\sin^2\theta$$

Figure 5 presents the result of this analysis for a circular cylinder in linearly sheared flows of various degrees. Frutiger[3] extended this analysis to elliptical cylinders by use of an elliptical coordinate system. Figures 6 and 7 show results for elliptical cylinders in linearly sheared flows.

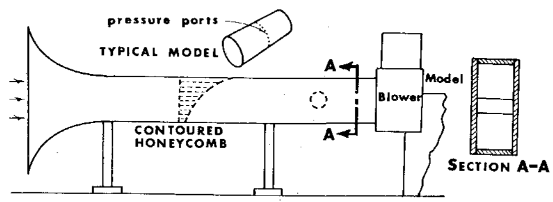

Fig. 11 Pressure measurement in sheared flow.

It will be noted that a symmetrical body in an unsymmetrically sheared field has an unsymmetrical pressure distribution. This pressure field will produce an effect similar to the Magnus effect, i.e., a lift force will be produced. It is well known that a plot of C_p vs chordwise station indicates the lift coefficient. Measurement of the areas between the C_p curves produces the lift effect tabulated below.

$$C_L \equiv L'/(\rho/2)V_0^2c$$

If the chord of each cylinder is 2 ft, and $V_0 = 100$ fps, then $K = 0.6$ gives an onset velocity equation of $u = 100 + 60y$; i.e., the rate of shear is 60 fps/ft.

Experimental Investigations

The experimental work consists of: generation of sheared flowfields in a special wind tunnel, and testing of pressure models of two-dimensional airfoils and cylinders.

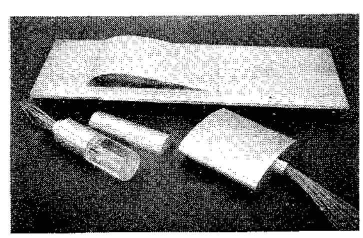

Fig. 12 Two-dimensional pressure models.

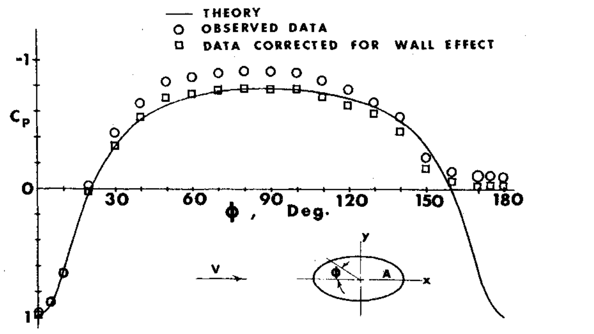

Fig. 13 Pressure distribution on 3–1 elliptical cylinder in uniform flow.

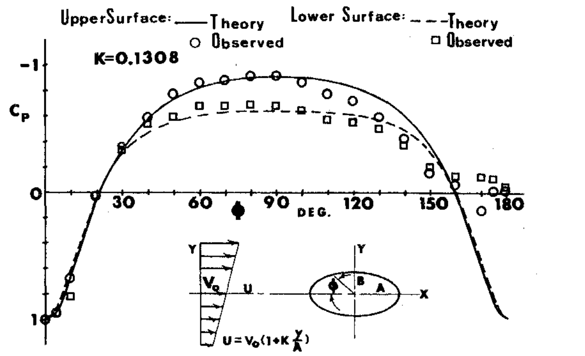

Fig. 15 3-1 elliptical cylinder in linearly sheared flow.

Shear Tunnel Tests

A small two-dimensional wind tunnel at Wichita State University was equipped to provide sheared flowfields as shown in Fig. 8. Kotansky[4] demonstrated that contoured honeycomb material could be used in a wind tunnel to produce sheared flow.

Colli[5] contoured a block of aluminium honeycomb to produce a typical turbulent boundary-layer velocity profile (one-seventh power law). The honeycomb used had a cell "diameter" of $\frac{3}{16}$ of an inch. The length of honeycomb block, $L(y)$ required to produce a given velocity distribution, $V(y)$ downstream of the block is given by the equation

$$L(y) = (d/4f)\ \{[V_0/V(y)]\}^2 - 1$$
$$1/(4f)^{1/2} = 2\log_{10}[Re(4f)^{1/2}] - 0.8$$

Figure 9 shows the block contour and the resulting velocity variations, and Fig. 10 is a photograph of some of the blocks used. Measurements of velocities and of turbulence level were made at four stations downstream of the block. The velocity distribution obtained was very close to the design distribution, was stable, and was fairly constant throughout the entire length of the test section. The levels of turbulence attained compared very well with values obtained from similar turbulent boundary layers produced naturally, although there is some decrease of turbulence level downstream.

Testing of Pressure Models

In order to test models, the tunnel was arranged as shown in Fig. 11 with the contoured honeycomb blocks filling the entire 6 in. × 14 in. cross section. The conditions reported include two linearly sheared flows, and one parabolic sheared flow. Models tested (see Fig. 12) included circular cylinder, 3-1 elliptical cylinder, circular cylinder (reflection plane), and Joukowski airfoil (reflection plane). Pressure data were taken over the surface of the models and converted to pressure coefficients based on tunnel centerline velocity. The size of the tunnel necessitated corrections for wall effects. Pope[6] recommends solid blockage and wake corrections. The correction for the uniform flow case was

$$C_{p_{cor}} = 0.9321 C_{p_{uncor}} + 0.0679$$

It can be seen from Fig. 13 that this correction does correct the raw data to produce reasonable agreement with the theoretical values. This same correction was used for the sheared flow cases.

The sheared flow results are shown in Figs. 13, 14, and 15. It will be noted that agreement with theory is good in the region of assisting pressure gradient. Downstream of the minimum pressure point the agreement is not as good, and, of course, there is no agreement beyond the separation point.

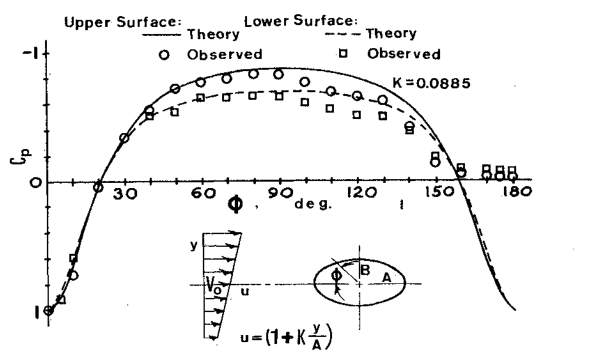

Fig. 14 Pressure distribution on 3–1 elliptical cylinder in linearly sheared flow.

Conclusions

It is possible to use potential flow methods to analyze the flow about objects in a sheared flowfield, except in cases where there is separation of the flow from the object.

References

[1] Monical, R. E. and Frutiger, L. D., "Drag of Objects in Boundary Layers," D3-7687, July 1968, The Boeing Co., Wichita, Kansas.

[2] Snyder, M. H., "Development of Automatic Computation of Drag of Objects in Sheared Flow Fields," Aeronautical Technical Note 68-004, July 1968, The Boeing Co., Wichita, Kansas.

[3] Frutiger, L. D., "The Effects of Shear Flow on the Pressure Distribution over Two-Dimensional Elliptical Cylinders," M.S. thesis, June 1968, Wichita State Univ., Wichita, Kansas.

[4] Kotansky, D. R., "The Use of Honeycomb for Shear Flow Generation," *AIAA Journal*, Vol. 4, No. 8, Aug. 1966, pp. 1490–1491.

[5] Colli, A., "Turbulent Boundary Layer Generation by the Use of Honeycomb in a Wind Tunnel," M.S. thesis, June 1969, Wichita State Univ., Wichita, Kansas.

[6] Pope, A., Chap. 6, *Wind Tunnel Testing*, 2nd ed., Wiley, New York, 1954.